UDC 539.3

Dynamic viscoelastic properties of isotropic composites with prestressed components

Maslov B.P.
S.P. Timoshenko Institute of Mechanics NAS Ukraine, Kyiv, Ukraine

Abstract. *Overall dynamic response and creep behavior of random multi-component composites with nonlinear constituents are investigated. As in Rabotnov's type theory, the behavior of the viscoelastic material is described by a quasi-linear law. The nonlinear function of instant elastic stress plays the role like that of the strain in the linear case. We use constitutive equations for statistical fluctuations of first and second order displacement, nonlinear Green deformation, nominal or Cauchy stress in the representative volume. Upon application of the integral Carson and Fourier transforms, the boundary value problem for the local stress and strain fields becomes similar to a linear elastic problem. The programs from NAG-Fortran library are used. The model suggested may be useful for long-term durability prediction under cyclic loading.*
Keywords. *viscoelastic composite;dynamic properties; cyclic loading.*

Introduction. On the basis of the nonlinear theory of viscoelasticity, the general constitutive equation for isotropic composites in the presence of initial stress is derived. Expressions for the nominal stress tensors in a finitely deformed configuration are given along with the elasticity tensors. The equations governing infinitesimal motions superimposed on a finite deformation are then used to study the effects of initial stress on the creep parameters in a homogeneously deformed, and initially stressed isotropic materials. The presence of initial stresses in solid materials is very different from the corresponding response in the absence of initial stresses. Some analogous problems in geophysics are very important too, the high stress developed due to gravity has a strong influence on the propagation speed of elastic waves etc.

Research objective. Superimposed on the equilibrium configuration, defined by $\boldsymbol{x} = \boldsymbol{\chi}(\boldsymbol{X})$, we now consider an incremental motion $\dot{\boldsymbol{x}}(\boldsymbol{X},t)$, where t is time. Here and in the following a superposed dot indicates an incremental quantity, increments are considered as small, and the resulting incremental equations are linearized in the increments

$$\bar{\boldsymbol{x}}(t) = \boldsymbol{x}(0) + \boldsymbol{u}(t), \quad \boldsymbol{u}(t) = \dot{\boldsymbol{x}}(t). \tag{1}$$

Thus, $\dot{\boldsymbol{x}}(t)$ represents the displacement from $\boldsymbol{x}(0)$ to current position, $\bar{\mathbf{x}}(\mathbf{X},t)$, and we shall also express it in Eulerian form by writing the displacement vector as a function of $\boldsymbol{x}$ and t, namely $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x},t)$. The corresponding increment in the deformation gradient, $\dot{\boldsymbol{F}} = \boldsymbol{H}$, is expressible as

$$\bar{\boldsymbol{F}} = \boldsymbol{F} + \dot{\boldsymbol{F}}; \ \dot{\boldsymbol{F}}(t) = \boldsymbol{\gamma}\boldsymbol{F}, \quad \boldsymbol{\gamma} = \partial \boldsymbol{u} / \partial \mathbf{x} \tag{2}$$

The linearized, incremental nominal, (Kirchhoff), stress $\dot{\boldsymbol{T}}$ takes the forms

$$\begin{gathered} \dot{\boldsymbol{T}}(\boldsymbol{X}) = \boldsymbol{t}(\boldsymbol{X}) = \boldsymbol{A}\boldsymbol{H}(\boldsymbol{X}); \ \ t^{\alpha i}(\boldsymbol{X}) = \mathrm{A}^{\alpha i \beta j}(\boldsymbol{X}) H_{j\beta}(\boldsymbol{X}); \\ \boldsymbol{\tau}(\boldsymbol{x}) = \boldsymbol{C}\boldsymbol{\gamma}(\boldsymbol{x}); \quad \tau^{pi}(\boldsymbol{x}) = C^{piqj}(\boldsymbol{x})\gamma_{jq}(\boldsymbol{x}); \ JC^{piqj} = F_{p\alpha}F_{q\beta}\mathrm{A}^{\alpha i \beta j}, \end{gathered} \tag{3}$$

If $\mathbf{X}$ be the position vector of a material point in reference configuration, then Div denote the gradient operator with respect to $\mathbf{X}$. We use Greek (Roman) characters for indices associated with the reference (current) configuration. $\mathcal{A} = \dfrac{\partial^2 W(\boldsymbol{F})}{\partial \boldsymbol{F}\, \partial \boldsymbol{F}}$ is the fourth-order elasticity tensor. The nominal $\boldsymbol{T}(\boldsymbol{X})$, and Cauchy $\boldsymbol{\sigma}(\boldsymbol{x})$ stresses are given by

$$\boldsymbol{T} = \frac{\partial W}{\partial \boldsymbol{F}}; \quad \boldsymbol{\sigma} = J^{-1}\boldsymbol{FT}. \tag{4}$$

The problem is to define correctly the equation of motion

$$Div\,\boldsymbol{t} = \rho_0 \boldsymbol{u},_{tt} \quad div\,\boldsymbol{\tau}(\boldsymbol{x}) = \rho \boldsymbol{u},_{tt}(\boldsymbol{x},t). \tag{5}$$

Then for representative volume of composite we obtain

$$\tilde{C}^{piqj}\left\langle u_j \right\rangle_{,qp} = \rho \left\langle u_i \right\rangle_{,tt} \tag{6}$$

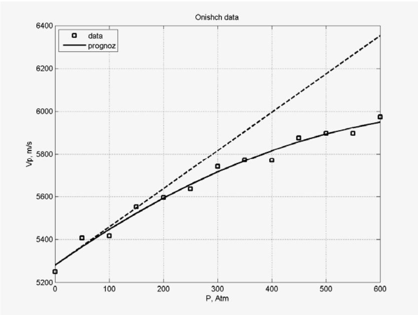

Fig. 1

A special case of the mechanics of materials is finite elastic deformations in which the strains are small but the linear theory is no longer adequate, so one may use second-order elasticity. In this theory, the strain energy function is expanded to the third order of strain and the stress is second order in the strain. It is the Green strain tensor that is used and for an isotropic material the strain energy is expressed in terms of strain invariants. We commence with the general tensorial form of quasilinear viscoelasticity (QLV) like that proposed by Rabotnov

$$\boldsymbol{\tau}^e(\boldsymbol{x},t) = \int_{-\infty}^{t} \boldsymbol{g}(t-t_1)\frac{\partial \boldsymbol{\tau}(\boldsymbol{x},t_1)}{\partial t_1}dt_1. \tag{12}$$

Here $\boldsymbol{g}(t)$ is the reduced stress relaxation fourth-order tensor, $\boldsymbol{\tau}(t)$ denotes the nominal stress, while $\boldsymbol{\tau}^e(\boldsymbol{e},t)$ is an instantaneous strain measure. The latter may be thought of as an equivalent (instantaneous) elastic stress. This tensorial integral identity is the natural generalization of the simple one-dimensional relationship, proposed by Rabotnov. We consider an elastic body that is subject to an initial stress $\mathbf{T}_i(\mathbf{X})$ in some reference configuration.

Incremental viscoelastic deformation of composites. The linearized theory of elasticity of pre-deformed bodies is increasingly used in solving important practical problems: elastic stability, wave propagation, etc. In this regard, it is necessary to substantiate the possibility of transferring the methods and results of this theory to heterogeneous viscoelastic media, including composites and geological materials. Although theory of pre-stressed materials associated with hyperelastic ones is well established, the analogous theory associated with QLV materials is not yet developed. We shall therefore derive such a theory. The viscoelasticity theory is extended in order to deal with materials that are subject to finite deformation, and whose constitutive response is nonlinear. In particular, a modified QLV theory is developed, where retardation is independent of stress.

$$\boldsymbol{\tau}(\boldsymbol{x},t) = \int_{-\infty}^{t} \boldsymbol{h}(t-t_1)\frac{\partial \boldsymbol{\tau}^e(\boldsymbol{x},t_1)}{\partial t_1}dt_1. \tag{13}$$

It follows that in the case of a uniform preliminary deformation of the equilibrium equation of the general theory, they coincide in form with the analogous equations of the classical theory of elasticity. The difference between this formulation and the linear theory is that the mean gradients of displacement increments $\left\langle H_{j\alpha} \right\rangle$ are of interest here, and not their symmetric combinations e_{ij}. And $H_{ja} = \gamma_{jm}F_{ma}$. Substituting this solution into the second-order averaged physical relations from, we find the relationship between the macroscopic nominal stresses, $\mathbf{T}$, and $\mathbf{H}_{[2]}$, the second approximation of gradients $\mathbf{H}$

$$\boldsymbol{T}_{[2]} = p_{[2]}\boldsymbol{I} + 2\mu\boldsymbol{e}_{[2]} - \left(p\boldsymbol{H} + \mu\boldsymbol{H}^2 + \gamma\boldsymbol{e}^2\right)_{[2]} \tag{14}$$

For a longitudinal wave in non-homogeneous geological media, velocity reduces to

$$\begin{gathered}\rho_0 V_P^2 = \lambda + 2\mu + \frac{1}{3}(7\lambda + 10\mu + 3\nu_1 + 10\nu_2 + 8\nu_3)E, \\ E = -\kappa / T_i\end{gathered} \tag{15}$$

As an example, on the Fig.1 there are the results of prediction of longitudinal velocity in viscoelastic non-homogeneous media under large preliminary stress. Firstly, we can see a significant difference between linear (dashed) and our (solid) prediction. Secondly, there is a rather good coincidence with experimental data (circles). Some examples of multi-component materials were modelled, elastic constants of which are presented in Table 1.

Table 1

Material	λ	μ	ν_1	ν_2	ν_3
Aluminium 2S	−204.0	27.6	−228.0	−197.0	−57.0
Pyrex glass	264.0	27.5	420.0	−118.0	105.0
SiO2 melted	72.0	31.3	−44.0	93.0	−11.0

Basic maintenance and results of research. The fractional exponential Rabotnov's type operator belongs to the class of well-studied resolvent operators. It has a number of properties used in decoding operator expressions [7]. In those cases, where knowledge of the exact result of the operator's action on a constant or variable value is required, we will use the Fortran F90 shell program package [12]. Using the correspondence principle, we analyze the dynamic problem for an isotropic composite material under a stationary, cyclic loading mode. If a viscoelastic material is affected by a periodic perturbation with frequency ω, then in (14) it is advisable to move to complex values $\mu(\omega) = \mu_R(\omega) + i\mu_1(\omega)$, where

$$\mu_R(\omega) = \mu\left[1 - \xi\beta^{-1}\frac{z\sin\frac{\pi\alpha}{2} + z^2}{1 + 2z\sin\frac{\pi\alpha}{2} + z^2}\right]; \quad \mu_1(\omega) = \mu\left[1 - \xi\beta^{-1}\frac{z\cos\frac{\pi\alpha}{2}}{1 + 2z\sin\frac{\pi\alpha}{2} + z^2}\right]; \quad z = \beta\omega^{\alpha-1}. \tag{16}$$

Dispersion and attenuation effects were evaluated with (15), (16) for composites and some geological structures. So, nonlinear dynamic viscoelastic problem is investigated in second order approximation theory when the gradient deformation terms higher than second order are neglected. Convex potential function or time-dependent functional are used to build up overall constitutive relations.

The criterion of long-term strength is the time limited increment of creep deformations. As an approximate, we use the criterion of critical deformation. Its essence lies in the fact that the critical time and critical forces are determined from the equality of the creep strain to that critical strain, which is calculated under the assumption that the viscoelastic body is deformed in an elastic region. Thus, the critical time is defined as the time required to achieve, at a given load, the creep strain of critical strain values for an elastic body. The most important extension of this work concerns using it to determine the second moment of viscoelastic stress increments. The developed method may find a variety of applications in the mechanics of materials, geophysics, etc.

Динамічні вязкопружні властивості ізотропних композитів із переднапруженими компонентами

Маслов Б.П.

Анотація. *Досліджено задачу прогнозування наведених динамічних властивостей і повзучості композитів з нелінійними компонентами. Згідно з теорією типу Работнова поведінка в'язкопружного матеріалу описується квазілінійним законом. Сформульовано розв'язуючі рівняння для флуктуацій переміщень першого і другого порядку малості. Інтегральними перетвореннями Карсона і Фур'є крайова задача зведена до аналогічної пружної. Запропонована модель може бути корисна для прогнозування довготривалої міцності при циклічному навантаженні.*
Ключові слова. *вязкопружні композити;динамічні властивості; циклічне навантаження.*

Динамические вязкоупругие свойства изотропных композитов с преднапряженными компонентами

Маслов Б.П.

Аннотация. *Исследована задача прогнозирования приведенных динамических свойств и ползучести композитов с нелинейными составляющими. Следуя теории типа Работнова, поведение вязкоупругого материала описывается квазилинейным законом. Сформулированы разрешающие уравнения для флуктуаций перемещений первого и второго порядка малости. Интегральными преобразованиями Карсона и Фурье краевая задача сведена к аналогичной задаче упругости. Предложенная модель может быть полезна для прогнозирования длительной прочности при циклическом нагружении..*
Ключевые слова. *вязкоупругие композиты; динамические свойства; циклическое нагружение.*

REFERENCES

1. Aboudi J. Micromechanics of Composite Materials / J.Aboudi, S.Arnold, B.Bednarcyk. – Elsevier. –2013. – 1011 p.
2. Biot M.A. Mechanics of incremental deformations // New York : John Willey and Sons. – 1965. – 504 p.
3. Parnell1 W. J. Soft metamaterials with dynamic viscoelastic functionality tuned by pre-deformation / W.J.Parnell, R.De Pascalis. – 2019. <hal-02012896>.
4. Гузь А.Н. Механика хрупкого разрушения материалов с начальными напряжениями. – Киев : Наук. думка, 1983. – 296 с.
5. Destrade M. On stress-dependent elastic moduli and wave speeds / M.Destrade, R.W.Ogden. // Journal of Applied Mathematics, 2013. – Vol. 78. – P. 965-997.
6. Maslov B.P. Corrected Constants for Composites with Initial Strains // Soviet Appl. Mech. – 1981. – Vol. 17, N9. – P. 815-820.
7. Maslov B.P. Reduced Dynamic Characteristics of Composite Materials with Initial Stress // Soviet Appl. Mech. – 1982. – Vol. 18, N6. – P. 547-550.
8. Golub V.P. Identification of the Hereditary Kernels of Isotropic Linear Viscoelastic Materials in Combined Stress State. II. Deviators Proportionality / V.P.Golub, B.P.Maslov, P.V.Fernati // International Appl. Mechanics, 2016, Vol. 52, N6. – P.111-125.
9. Maslov B.P. Numerical-analytical determination of Poisson's ratio for viscoelastic isotropic materials // International Applied Mechanics. – 2018. – Vol. 54, N2. – P.113-124.
10. Maslov B. Stress concentration in nonlinear viscoelastic composites // Mechanics and Advanced Technologies, 2017. – N1 (79). – P. 5-10.

Научные исследования, результаты которых опубликованы в данной статье, выполнены за счет средств бюджетной программы «Поддержка приоритетных направлений научных исследований» (КПКВК 6541230).