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MECHANICAL SIMULATION OF THE MOTION OF GEOSTATIONARY SATELLITES

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Abstract. It is researched influence of inertia force as an internal volumetric potential force on the nature of the displacement of the center of mass of a solid body (artificial satellite) in a geostationary orbit. It is developed mechanism to simulate the movement of a satellite in a geostationary orbit, which contains rotating disks of various diameters and provides uniform motion of the axis of rotation of a small-diameter disk around a larger-diameter disk with equal angular velocities of rotation of these disks. Moreover, the mechanism provides the possibility of rotation of the disk of small diameter both in the direction of rotation of the disk of large diameter, and in the opposite direction. In both cases, the total kinetic energy of the small disk remains constant, but the energy consumption of the sources of forces that provide each of these movements change. The established relationship for geostationary satellites of various designs and purposes remains the same.
Keywords: solid body; inertia force; gravitational field; satellite; mechanism.

Introduction. The research of theoretical calculations of processes and phenomena often consist of the improvement of various models made for the mathematical description of the object of study [1]. Among the variety of existing models, mechanical models occupy an important place, which make it possible to assess the correctness of the assumptions made for theoretical analysis of the object of study[1].

Purpose of work. Using the basic ideas about the motion of a rigid body in a geostationary orbit, justify and develop a mechanical model for studying the characteristic features of the motion of geostationary satellites.

It is shown in [3] that the gravitational field of the Earth could be represented as the result of the interaction of two potential fields: the displacement field and the velocity field. It is because any material body (material point) represents object, the volume of which can be less than any previously given small volume. The point of application of external force to the material body is a geometric point, the dimensions of which are always smaller than the dimensions of the material body [3]. That is, within each body, internal forces interact with external forces applied to the body.

The balance of power when body m is falling in the stationary gravitational field of the Earth (Fig. 1 *a*) without taking into account the resistance of the atmosphere is determined by the equation

$$G_F(r) - \Phi_Q^{(i)}(r) = 0\,, \tag{1}$$

where $\Phi_Q^{(i)}(r) = -mg(r)$[2] - the force of inertia of the body m, which is the internal volumetric potential resistance force[3] [3], counteracting external force of motion - gravity force $G_F(r) = mg(r)$ on a distance r from the center of mass of the Earth.

The magnitude of the gravity of the body m in the stationary force field of the Earth (Fig.1) is determined by the Newton's law of universal gravitation in the form of a function [4]

$$U_m(r) = G_0 \frac{Mm}{r^2} = mg(r) = G(r)\,, \tag{2}$$

[1] Sir William Thomson, 1st Baron Kelvin «I can say that I understood the phenomenon, if I can create a mechanical model for it» [2].

[2] Hereinafter, the superscript is used in the notation of internal force (*i*). Lowercase letters *F* and *Q* separated by a comma from other letters and numbers of the lower index indicate the nature of the force (driving forces - *F*, motion resistance forces - *Q*).

where $U_m(r)$ - function determining the potential of the body m in a stationary force field of the Earth; $G_0 = 6{,}6743 \cdot 10^{-11}\ m^3 / s^2 kg$ - gravitational constant; $g(r)$ - acceleration of

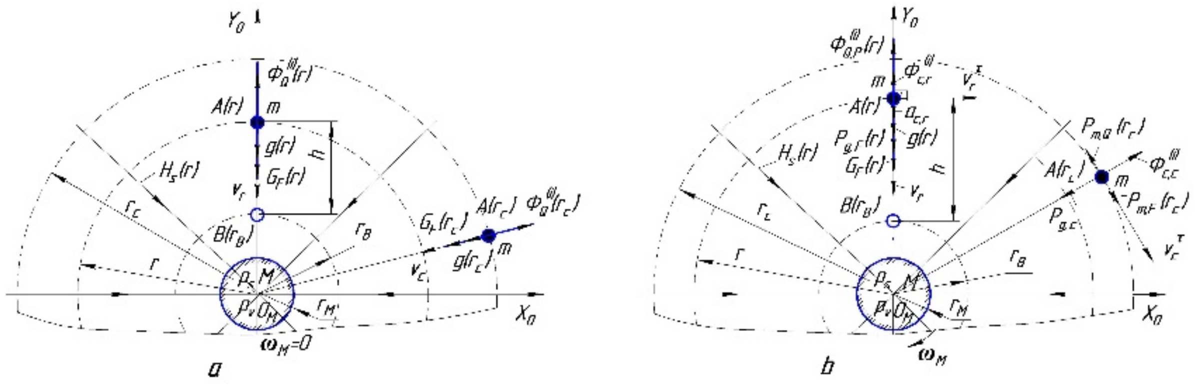

Fig. 1. Steady movement of the body m in the gravitational field of the Earth: *a* – in the stationary gravitational field; *b* - during the rotation of the gravitational field

gravity of body m in a potential force field of the Earth.

It was shown in [3] that the potential velocity field always corresponds to the potential velocity field with a pole P_v in the center of mass M for which

$$\nabla(V_m(r)) = grad(V_m(r)) = -grad(U_m(r)) = -\nabla(U_m(r)), \quad (3)$$

where $V_m(r) = \dfrac{T(r)}{r} = \dfrac{mv_r^2}{2r} = \Phi_Q^{(i)}(r) = -mg(r)$ - the potential of the velocity field for the body m in the stationary gravitational field of the Earth, which determines the value of the inertia force at the point $A(r)$ of the force velocity field (Fig. 1 *a*).

From the condition of conservation of mechanical energy with a free fall of a body m without atmospheric resistance from a height $h = r - r_B$ (Fig. 1 *a*) follows [5]

$$E_m(r) = \Pi(r) + T(r) = \Pi(r_B) + T(r_B) = E_m(r_B) = \Pi(r_c) + T(r_c) = E_m(r_c), \quad (4)$$

where $E_m(r)$ and $E_m(r_B)$, $E_m(r_c)$; $\Pi(r) = mg(r)r$ and $\Pi(r_B) = mg(r_B)r_B$, $\Pi(r_c) = mg(r_c)r_c$; $T(r) = \dfrac{mv_r^2}{2}$ and $T(r_B) = \dfrac{mv_{r_B}^2}{2}$, $T(r_c) = \dfrac{mv_c^2}{2}$ - respectively, total mechanical energy, potential energy, kinetic energy of the body m in the points $A(r)$ and $B(r_B)$, $A(r_c)$ of the gravitational field of the Earth (r_c - radius of the geostationary orbit).

When the body m falls in the gravitational field of the Earth rotating with an angular velocity ω_M (Fig. 1 *b*), centripetal accelerations $a_{c,r} = \dfrac{(v_r^\tau)^2}{2} = \dfrac{(\omega_M r)^2}{2}$ and $a_{c,c} = \dfrac{(v_c^\tau)^2}{2} = \dfrac{(\omega_M r_c)^2}{2}$, respectively, act on the body m at the points $A(r)$ and $A(r_c)$, which determine the action of centrifugal inertia forces $\Phi_{c,r}^{(i)} = -ma_{c,r}$ and $\Phi_{c,c}^{(i)} = -ma_{c,c}$, providing the condition for the balance of forces at the points $A(r)$ and $A(r_c)$ [6]:

$$P_{g,F}(r) = G_F(r) - \Phi_{c,r}^{(i)} = -(\Phi_Q^{(i)}(r) - \Phi_{c,r}^{(i)}) = -m(g(r) - a_{c,r}) = -\Phi_{Q,P}^{(i)}(r), \quad (5)$$

$$P_{g,c} = G_F(r_c) - \Phi_{c,c}^{(i)} = 0, \quad (6)$$

where $P_{g,F}(r) \neq 0$ and $P_{g,c} = 0$ - body m weight on the equipotential surface of the rotating gravitational field of the Earth, which determines the potential energy of the body $\Pi^{/}(r) = P_{g,F}(r)r \neq 0$ and $\Pi^{/}(r_c) = 0$, when the linear velocity of rotation of the body m is equal to the first cosmic velocity $v_c = 3{,}072\ m/s$ [6].

From (4), (6), it follows:

$$E_m(r_c) = \Pi(r_c) + T(r_c) = 2T(r_c) = E_m^{/}(r_c) = \Pi^{/}(r_c) + T^{/}(r_c) = T^{/}(r_c) = E_m^{/}(r_c), \tag{8}$$

$$T^{/}(r_c) = 2T(r_c) = mv_c^2. \tag{9}$$

It is shown in Fig. 2 mechanism simulating the motion of a body m at a geostationary orbit. The two-link mechanism contains a disk 1_a with radius r_1 pivotally mounted on the axis of rotation 0 and is rigidly (as a whole body) connected with the carrier 1_b with length $l = r_1 + r_2$. The disc 2 is pivotally mounted at the end of the carrier 1_b, the axis of rotation of which describes the trajectory

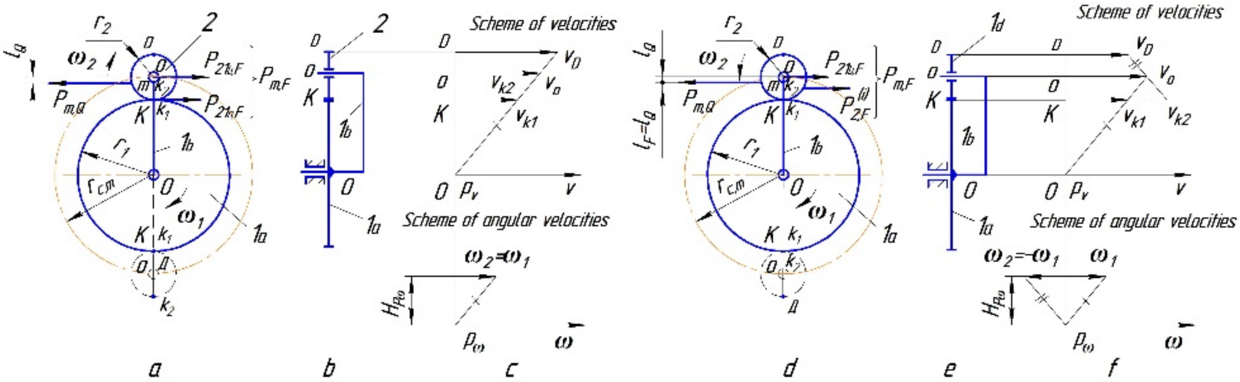

Fig. 2. The mechanism for modeling the motion of a rigid body in a geostationary orbit: *a, b, d, e* - schemes of the mechanism; *b, f* - schemes of linear and angular velocities of the links of the mechanism

(circle with radius $r_{c,m} = l_b$), which simulates the geostationary orbit along which the body m (disk 2) moves, when $r_2 << r_{c,m} \approx r_1$ and the angular velocity ω_1 and it also models the angular velocity of the rotating gravitational field of the Earth.

Depending on the contact conditions of the disks 1_a and 2 at the point (K) of their interface, the two-link mechanism can structurally represent two different mechanisms, each of which determines the distribution of energy-force parameters that provide different conditions for the disk 2 to move around the circle with a radius $r_{c,m} = l_b$.

In Fig. 2 *a, b, c* it is shown a mechanism for which (Fig. 2 *a*) in the interaction point K of the disks 1_a and 2 there is no slipping of the points $k1$ (disk 1_a) and $k2$ (disk 2). Analysis of the schemes of linear and angular velocities (Fig. 2 *b, c*) [7] and the trajectories of the points $k1$ and $k2$ also shows. The disk 2 rotates in the direction of rotation of the disk 1_a at an angular speed $\omega_2 = \omega_1$ the same points $k1$ and $k2$, which at the beginning of the revolution were at the mating point K, after making one revolution of the disks 1_a and 2 are again aligned at the K. The similarly moves the «satellite-wheel» in a geostationary orbit [8]. The kinetic energy of the disk

$$T(r_{c,m}) = \frac{mv_0^2}{2} + \frac{J_2\omega_2^2}{2} = mv_0^2. \tag{10}$$

where $J_2 = mr_2^2$ - moment of inertia of the disk 2 in the form of a heavy disk and $v_0 = \omega_2 r_2 = \omega_1 r_{c,m}$. The condition for uniform rotation of the disk provides a diagram of the forces applied to the disk 2. The forces of motion $P_{21b,F}$ and $P_{21a,F}$ act on the disk 2 from the side of the link 1 (disk 1_a and carrier 1_b) and the simulate influence of the rotating gravitational field of the Earth on the «satellite-wheel». The resistance force $P_{2,Q}$ acts on the disk from the side of stationary field of the Earth, which simulates the resistance force $P_{2,Q}$ to move the «satellite-wheel».

$$\Sigma P = P_{21b,F} + P_{21a,F} - P_{2,Q} = 0 \text{ and } \Sigma M = P_{21b,F} r_2 - P_{2,Q}(r_2 - l_Q) = 0. \quad (11)$$

Therefore, the «satellite-wheel» movement is provided by the work of the forces of the rotating gravitational field of the Earth, which is equal to the kinetic energy of the satellite-wheel ($T^{/}(r_c)$), and additional energy is not required for the motion of the «satellite-wheel».

In Fig. 2 *d*, *e*, *f* it is shown a mechanism for which (Fig. 2 *d*) there is a displacement without friction of the disk 2 relative to the disk 1_a (point $k2$ relative to the point $k1$) at the point K of contact of the disks 1_a and 2. An analysis of the schemes of linear and angular velocities shows that this condition is met when $\omega_2 = -\omega_1$, when the same point $k2$ is constantly at the conjugation point K of the rotating disks, due to the interaction of forces $P_{21b,F}$, $P_{2,F}^{(i)}$ and $P_{2,Q}$

$$\Sigma P = P_{21b,F} + P_{2,F}^{(i)} - P_{2,Q} = 0 \text{ and } \Sigma M = P_{2,F}^{(i)} l_Q - P_{21b,F} l_Q = 0, \quad (12)$$

where $P_{2,F}^{(i)} = 0{,}5(P_{21b,F} + P_{21a,F})$ is the internal force simulating the reactive thrust of the geostationary satellite's energy source, which is equal to half the motion forces of the rotating gravitational field of the Earth acting on the satellite.

A change in the direction of the angular velocity of the disk 2 (Fig. 2 *e, f*) does not lead to the kinetic energy change, which simulates the motion of a geostationary satellite without position change relative to the rotating surface of the Earth. This movement of the satellite must have a source of energy, the work of which is at least half of the kinetic energy of the «satellite-wheel».

Conclusion. A mechanical model of motion of geostationary satellites of various design and purpose is developed. Analysis of the mechanical model allows us to estimate the energy consumption associated with the operation of satellites in the geostationary orbit.

References:

1. Abashkov V.P. Klassifikaciya metodov modelirovaniya processov obrabotki metallov davleniem. [Classification of modeling methods for metal forming processes] / V.P. Abashkov, K.N. Solomonov // Universities Proceedings. Ferrous metallurgy. - 2008. - No. 9. - pp. 25–28.
2. Mandelstam L.I. Lekcii po teorii kolebanij. [Lectures on the theory of oscillations] / L.I. Mandelshtam- M.: "Science", 1972. - 470 p.
3. Dobrov I.V. Development of Scientific Bases of the Dynamics of Machines as a Section of Applied Mechanics / I.V. Dobrov // Procedia Engineering. – 2015. – V 129. – pp. 863–872.
4. Newton I. The Principia, Mathematical principles of natural philosophy / I. Newton. – London: Printed for Benjamin Motte, 1729. – 688 p.
5. Loitsyansky L.G. Kurs teoreticheskoj mekhaniki [The course of theoretical mechanics: in 2 volumes] / L.G. Loytsyansky, A.I. Lurie. - M.: Drofa, 2006 .- 719 p. – V 2.
6. Geostationary orbit. Available via DIALOG. https://en.wikipedia.org/wiki/Geostationary_orbit. Accessed 30 Jan 2020.
7. Kozhevnikov S. N. Teoriya mekhanizmov i mashin. [Theory of mechanisms and machines]. 4th ed., revised and added. / S. N. Kozhevnikov - Kiev: Mashgiz, 1973.- 592 p.

8. Ilyin A.A. Obespechenie orientacii malogo sputnika, stabiliziruemogo sobstvennym vrashcheniem. [Ensuring orientation of a small satellite stabilized by its own rotation] / A.A. Ilyin, M.Yu. Ovchinnikov, V.I. Penkov - M.: Preprint named after M.V. Keldysh RAS, 2004. - No. 83. - pp. 3-28.

МЕХАНІЧНЕ МОДЕЛЮВАННЯ РУХУ ГЕОСТАЦІОНАРНИХ СУПУТНИКІВ

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Анотація. Показано вплив сили інерції як внутрішньої об'ємної потенційної сили на характер переміщення центру маси твердого тіла (штучного супутника) на геостаціонарній орбіті. Розроблено механізм моделювання переміщення супутника на геостаціонарній орбіті, який містить обертові диски різного діаметру і забезпечує рівномірний рух осі обертання диска малого діаметра навколо диска більшого діаметра при рівних величинах кутових швидкостей обертання цих дисків. При цьому механізм забезпечує можливість обертання диска малого діаметра як в сторону обертання диска великого діаметра, так і в протилежну сторону. В обох випадках сумарна кінетична енергія малого диска залишається постійною, але змінюються витрати енергії джерел сил, що забезпечують кожне з цих рухів. Встановлена закономірність буде справедливої і для геостаціонарних супутників різної конструкції і призначення.

Ключові слова: *тверде тіло, сила інерції, гравітаційне поле, супутник, механізм.*

МЕХАНИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДВИЖЕНИЯ ГЕОСТАЦИОНАРНЫХ СПУТНИКОВ

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Аннотация. Показано влияние силы инерции как внутренней объемной потенциальной силы на характер перемещения центра массы твердого тела (искусственного спутника) на геостационарной орбите. Разработан механизм моделирования перемещения спутника на геостационарной орбите, который содержит вращающиеся диски различного диаметра и обеспечивает равномерное движение оси вращения диска малого диаметра вокруг диска большего диаметра при равных величинах угловых скоростей вращения этих дисков. При этом механизм обеспечивает возможность вращения диска малого диаметра как в сторону вращения диска большого диаметра, так и в противоположную сторону. В обоих случаях суммарная кинетическая энергия малого диска остается постоянной, но меняются затраты энергии источников сил, обеспечивающих каждое из этих движений. Установленная закономерность будет справедливой и для геостационарных спутников различной конструкции и назначения.

Ключевые слова: *твердое тело, сила инерции, гравитационное поле, спутник, механизм.*