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About possible displacements of a rigid body during rotation around a fixed axis displaced relatively to the center of mass

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Abstract. *Based on the analysis of virtual geometric displacements when solving a kinematic problem of Applied Mechanics, representing the uniform rotation of a free rigid body around a fixed vertical axis displaced relative to the center of mass of this body. Necessary and sufficient conditions have been established for such motion in the form of portable rotation of the center of mass of the body in a circle and relative motion of the body in the form of its rotation relative to the center of mass.*
Keywords: *rigid body, center of mass, kinematics, circular motion, virtual movements, translational rotation, relative rotation.*

Introduction. The issue of uniform motion of a rigid body in the form of a material point in a circle around a fixed axis displaced relative to the material point is the subject of special research in one of the sections of kinematics and is considered to be thoroughly studied [1]–[3]. However, the representation of a body in the form of a material point, which has no geometric dimensions, excludes the possibility of rotation of this body (material point) relative to the center of mass during any motion of the body.

Purpose of the study. Analysis of the kinematics of rigid bodies, the geometric dimensions of which are finite and can be less than any small predetermined size of space [4] in which a rigid body moves in a circle with a displaced fixed center relative to the center of mass of this body by the methods of Applied Mechanics.

Main part. Theoretical studies are carried out on the basis of the application of the basic provisions of the theory of mechanisms and machines (Applied Mechanics) for the kinematic analysis of a two-rod mechanism that implements the free motion of a rigid body in the form of a disk, the center of which is displaced relative to the center of mass of the disk [5].

The kinematic diagram of such a movement is presented in Fig. 1 and contains: a disk with a diameter d, mounted in a perfect hinge on a horizontal rod 2 at a distance l_{Oo} from a fixed axis of rotation OO a vertical rod, which forms an inseparable connection with the horizontal rod, and the vertical rod 3 itself is mounted on a fixed support 4 by a revolute pair and can rotate with an angular velocity in the clockwise direction.

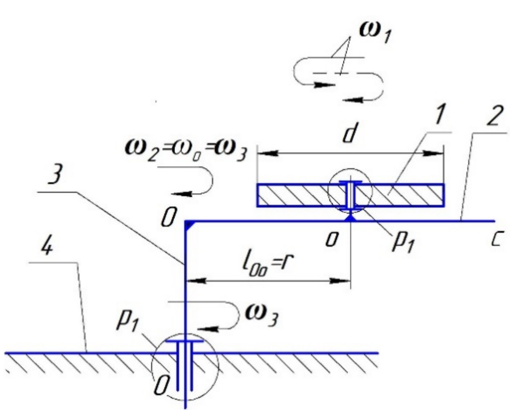

Fig. 1.

Thus, this kinematic scheme contains two moving links (n = 2) relative to the fixed support 4: one link in the form of a freely mounted disk 1 and the other link in the form of a lever 2, consisting of two inseparably connected rods 2 and 3; as well as two kinematic pairs of the first kind ($p_1 = 2$) in the form of revolute pairs p_1.

The degree of mobility (w) of such a mechanism according to the Chebyshev equation [5], [6] is:

$$w = 3 \cdot n + 2 \cdot p_1 = 3 \cdot 2 - 2 \cdot 2 = 2 . \qquad (1)$$

From equation (1) it follows that each of the two moving links of the mechanism can perform only one of the independent movements in the horizontal plane. The main (translator) movement is in the form of uniform rotation of the leading link of the mechanism - a lever with a point (center of mass of disk 1) located on its horizontal rod with an angular velocity $\omega_2 = \omega_3 = \omega_o$ in the clockwise direction of rotation and only one relative movement of the driven link of disk 1 with uniform rotation relative to the current position of the center of mass on the wheel ω_1 from the total number of virtual uniform angular velocities $\omega_{1,i}$ rotation of the free disk 1 relative to its center of mass in the direction of the rotational velocity ω_o

$$\overset{n}{\underset{i=1,2\dots}{\forall}} (\omega_{1,i} \geq \omega_o), \tag{2}$$

or in the opposite direction of the angular velocity ω_o:

$$\overset{n}{\underset{i=1,2\dots}{\forall}} (\left|-\omega_{1,i}\right| \leq \omega_o). \tag{3}$$

Analysis of equations (2), (3) shows that the conditions of equation (2) represent only virtual possible relative movements of disk 1, which in Fig. 1 are marked with dash-dotted lines and cannot be its real movements, since the angular velocity of the driven link of disk 1 in a mechanism with two degrees of freedom of its links under no circumstances can be equal to or even exceed the angular velocity of rotation of the driving link [5]. Therefore, virtual movements according to equation (2) must be excluded from the kinematic analysis of virtual relative movements of disk 1 during finding the real relative movement [7].

In turn, the analysis of equation (3) can be presented as a kinematic analysis of two equations:

$$\overset{n}{\underset{i=1,2\dots}{\forall}} (\left|-\omega_{1,i}\right| < \omega_o); \tag{4}$$

$$\overset{n}{\underset{i=1,2\dots}{\forall}} (\left|-\omega_{1,i}\right| = \left|-\omega_1\right| = \omega_o). \tag{5}$$

The relationship between the kinematics of the transfer and relative rotations of the disk 1 according to equations (4), (5) can be presented on the example of one revolution of the horizontal rod 2 (the center of mass of the disk 1 around a circle with radius $r = l_{Oo}$) respectively in Fig. 2 and Fig. 3.

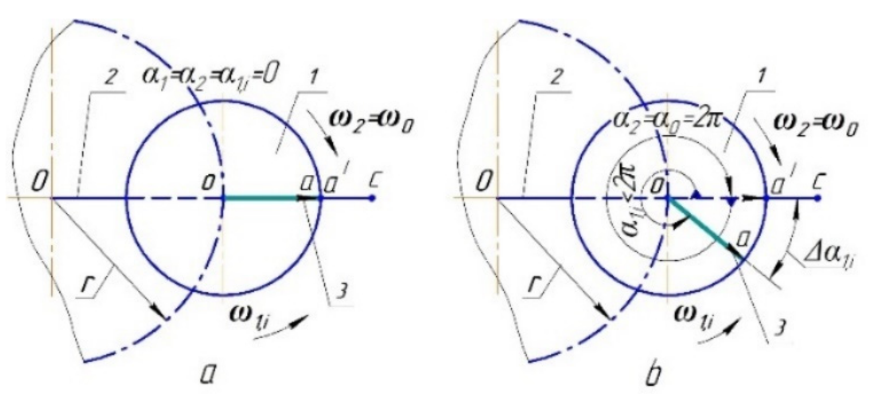

Fig. 2

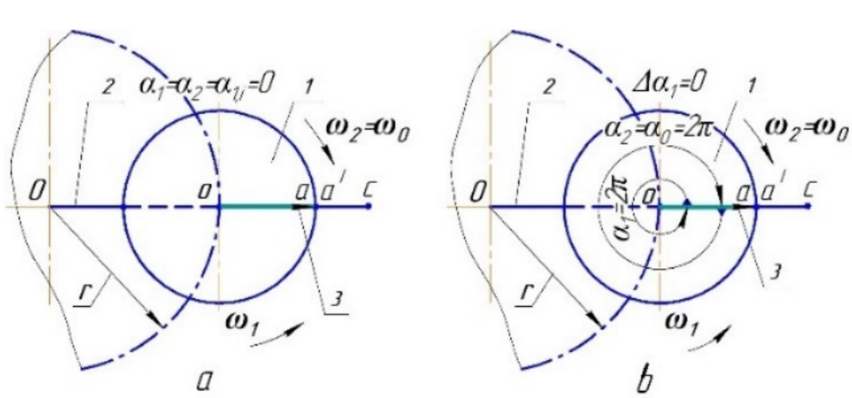

Fig. 3.

Let us at the initial analyzed time point $t = t_0$ uniform rotation of horizontal rod 2 with angular velocity ω_2 (Fig. 2, *a* and Fig. 3 *a*) radial arrow 3 (segment oa) on the surface of disk *1* will be directed along rod 2 in the direction of the segment $Oa^{/}c$ from point O to point c . During the time $t = T$ period of rotation of the rod *2* with angular velocity ω_2 and, accordingly, the period of rotation of a point around a circle with radius $r = l_{Oo}$ with angular translational velocity $\omega_0 = \omega_2$ rotation of rod 2 relative to the center O and motion of the point o around circle r determine the angles of rotation $\alpha_2 = \omega_2 T = \alpha_0 = \omega_0 T = 2\pi$ (Fig. 2, *b* and Fig. 3 *b*). In this case, the direction of the rod 2 and point position o on a circle r will not change with respect to their initial positions when $t = 0$ (Fig. 2, *a* and Fig. 3 *a*). In this case, if we do not consider the rotation of disk 1 with a relative angular velocity $-\omega_{1,i}$ (Fig. 2, *b*) or $-\omega_1$ (Fig. 3, *b*), then the radial arrow oa from its starting position also turn to the corner 2π and coincide in the direction of the arrow $oa^{/}$ on a segment $Oa^{/}c$ (Fig. 2, *b* and Fig. 3 *b*). However, during $t = T$ disk *1* rotating relative to the current point position o with relative angular velocity $\left|-\omega_{1,i}\right| < \omega_0$ will turn with the corner $\left|-\alpha_{1,i}\right| = \left|-\omega_{1,i}\right|T < \alpha_0 = 2\pi$ and will take the position determined by the radial arrow oa relative to the segment $Oa^{/}c$ (Fig. 2, *b*). That is, at any relative velocity from the number of possible angular velocities according to equation (4), at each rotation of the center of mass of the free disk 1 (point o) around a circle with a translational speed ω_0 you can fix the rotation of the disk *1* (radial arrow oa) relative to the lever *2* (relative to the point o) at an angle $\Delta\alpha_{1,i} = -\omega_{1,i}T + 2\pi > 0$ for each of the possible angular velocities $-\omega_{1,i}$ according to the equation (4). One such movement contradicts the condition arising from equation (1), which determines the possibility of relative rotation of the disk *1* with only one uniform angular velocity in magnitude and direction. Thus, the condition arising from equation (1) will be fully satisfied only by the relative angular velocity $-\omega_1 = \omega_0$ from the equation (5) at which (Fig. 3, b) angle of relative rotation of the disk 1 (radial arrow oa from the position $oa^{/}$) is equal to $\left|-\alpha_1\right| = \left|-\omega_{1,i}\right|T = \alpha_0 = 2\pi$ and the relative rotation angle is $\Delta\alpha_1 = -\omega_1 T + 2\pi = 0$. That is, the position at which the relative rotation of the disk *1* relative to the rod *2* makes an angle 2π.

The results of theoretical studies were confirmed by experimental studies performed using a digital movie camera mounted above the axis of rotation of the vertical rod. 3 (in Fig. 1 this camera is conditionally not shown) to record the process of rotation in a circle with a radius $r = l_{Oo}$ of the freely installed disk *1*, on the surface of which a radial arrow is applied. The results of the shooting were displayed on the monitor of a personal computer in online mode [8].

Conclusions

Based on the conducted theoretical and experimental studies of the free rotation of a symmetric body in a circle around a fixed vertical axis displaced relative to the center of mass, it was established that this process determines both the movement of the center of mass in a circle with a translational angular velocity, and the rotation of the body around the center of mass with a relative angular velocity, the magnitude of which is equal to the translational angular velocity, and the direction of the relative angular velocity is opposite to the direction of the translational angular velocity of rotation of the center of mass in a circle.

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Про можливі переміщення твердого тіла при обертанні навколо нерухомої осі, зміщеної щодо центра маси

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Анотація. *На підставі аналізу віртуальних геометричних переміщень при вирішенні кінематичної задачі прикладної механіки, що представляє рівномірне обертання вільного твердого тіла навколо вертикальної нерухомої осі, зміщеної щодо центру маси цього тіла. Встановлено необхідні та достатні умови для такого руху у вигляді переносного обертання центру маси тіла по колу та відносного руху тіла у вигляді його обертання щодо центру мас.*
Ключові слова: *тверде тіло, центр мас, кінематика, круговий рух, віртуальні переміщення, переносне обертання, відносне обертання.*